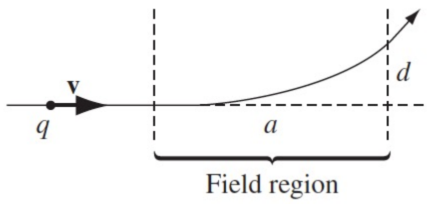
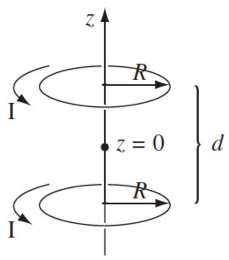
PHY209 Electromagnetism
Assignment 8
Handed out: November 5, 2019

Problem 1



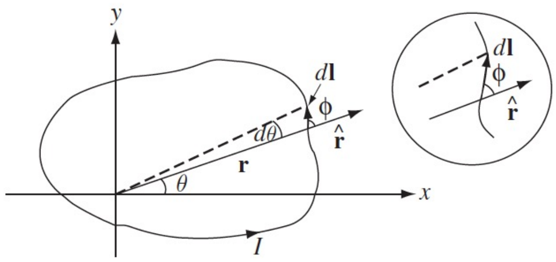
(a) A charge q enters a region of uniform magnetic field $\boldsymbol{B}$ (pointing into the page). The field deflects the charge a distance d above the original line of flight, as shown in Fig. What is the sign of the charge? In terms of a, d, B and q, find the momentum of the charge.
(b) Suppose, for instance, that $\boldsymbol{B}$ points in the x-direction, and $\boldsymbol{E}$ in the z-direction. A positive charge is released from the origin with $\boldsymbol{v} = \frac{E}{B}\hat{\boldsymbol{y}}$; what path will it follow?
(c) In 1897, J. J. Thomson discovered the electron by measuring the charge-to-mass ratio of cathode rays (actually, streams of electrons, with charge q and mass m) as follows:
(i) First he passed the beam through uniform crossed electric and magnetic fields $\boldsymbol{E}$ and $\boldsymbol{B}$ (mutually perpendicular, and both of them perpendicular to the beam), and adjusted the electric field until he got zero deflection. What was the speed of the particles (in terms of E and B)?
(ii) Then he turned off the electric field, and measured the radius of curvature, R, of the beam, as deflected by the magnetic field alone. In terms of E, B, and R, what is the charge-to-mass ratio ($\frac{q}{m}$) of the particles?

Problem 2



The magnetic field on the axis of a circular current loop is far from uniform (it falls off sharply with increasing z). You can produce a more nearly uniform field by using two such loops a distance d apart (Figure).
(a) Find the field (B) as a function of z, and show that $\frac{\partial B}{\partial z}$ is zero at the point midway between them ($z = 0$).
(b) If you pick d just right, the second derivative of B will also vanish at the midpoint. This arrangement is known as a Helmholtz coil; it's a convenient way of producing relatively uniform fields in the laboratory. Determine d such that $\frac{\partial^2 B}{\partial z^2} = 0$ at the midpoint, and find the resulting magnetic field at the center.

Problem 3



Consider a plane loop of wire that carries a steady current I; we want to calculate the magnetic field at a point in the plane. We might as well take that point to be the origin (it could be inside or outside the loop). The shape of the wire is given, in polar coordinates, by a specified function $r(\theta)$ (Figure).

(a) Show that the magnitude of the field is

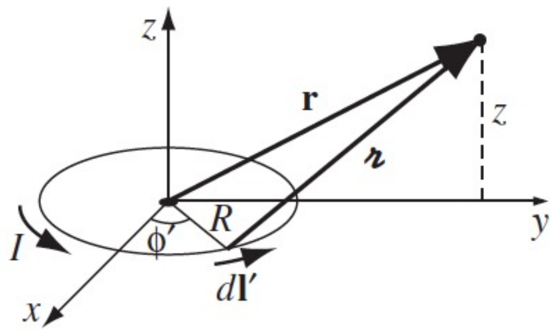
$$B = \frac{\mu_0 I}{4\pi} \oint \frac{d\theta}{r} \tag{1}$$

(b) Test this formula by calculating the field at the center of a circular loop.

(c) The lituus spiral is defined by $r(\theta) = \frac{a}{\sqrt{\theta}}$, $(0 < \theta < 2\pi)$ (for some constant a). Sketch this figure, and complete the loop with a straight segment along the x axis. What is the magnetic field at the origin?

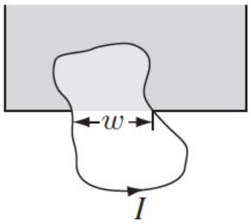
(d) For a conic section with focus at the origin, $r(\theta) = \frac{p}{1+e\cos\theta}$, where p is the y intercept and e is the eccentricity ($e = 0$ for a circle, $0 < e < 1$ for an ellipse, $e = 1$ for a parabola). Show that the field is $B = \frac{\mu_0 I}{2p}$ regardless of the eccentricity.

Problem 4



Suppose you wanted to find the field of a circular loop at a point $\boldsymbol{r}$ that is not directly above the center (Figure). You might as well choose your axes so that $\boldsymbol{r}$ lies in the yz plane at $(0, y, z)$. The source point is $(R\cos\phi, R\sin\phi, 0)$, and ϕ runs from 0 to 2π. Set up the integrals from which you could calculate B_x, B_y, and B_z, and evaluate B_x explicitly. In the $y \gg R$ limit (that is, very far from the ring), make suitable approximations and show that the magnitude of the magnetic field for a point on the y axis is approximately equal to $(\frac{\mu_0}{4\pi})(\frac{m}{a^3})$, where $m = \pi R^2 I$ is the magnetic dipole moment of the ring. Ignore the very small $\frac{R^2}{y^2}$ term and use $(1+\epsilon)^{-\frac{3}{2}} \approx 1 - 3/2\epsilon$.

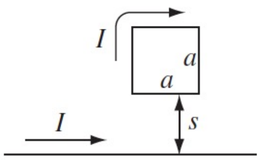
Problem 5



A plane wire loop of irregular shape is situated so that part of it is in a uniform magnetic field $\boldsymbol{B}$ (in Figure the field occupies the shaded region, and points perpendicular to the plane of the loop). The loop carries a current I. Show that the net magnetic force on the loop is $F = IBw$, where w is the chord subtended. What is the direction of the force?

Problem 6

(a) Suppose that the magnetic field in some region has the form $\boldsymbol{B} = kz\hat{\boldsymbol{x}}$ (where k is a constant). Find the force on a square loop (side a), lying in the yz plane and centered at the origin, if it carries a current I, flowing counterclockwise, when you look down the x axis.
(b) Find the magnetic field at the center of a square loop, which carries a steady current I. Let R be the distance from center to side.
(c) Find the force on a square loop placed as shown in Figure, near an infinite straight wire.



Both the loop and the wire carry a steady current I.
(d) Find the exact magnetic field a distance z above the center of a square loop of side a, carrying a current I . Verify that it reduces to the field of a dipole, with the appropriate dipole moment, when $z \gg a$.
(e) Calculate the torque exerted on the square loop shown in Figure, due to the circular loop

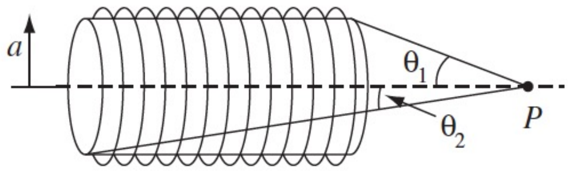
(assume r is much larger than a or b). If the square loop is free to rotate, what will its equilibrium orientation be?

Problem 7

(a) Find the field at the center of a regular n-sided polygon, carrying a steady current I. Again, let R be the distance from the center to any side.

(b) Check that your formula reduces to the field at the center of a circular loop, in the limit $n \to \infty$.
(c) Let $\boldsymbol{A}(r, \theta, \phi) = \frac{\mu_0}{4\pi}\frac{\boldsymbol{m}\times\hat{\boldsymbol{r}}}{r^2}$. Show that $\boldsymbol{B}(r, \theta, \phi) = \nabla \times \boldsymbol{A} = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\hat{\boldsymbol{r}} + \sin\theta\hat{\boldsymbol{\theta}})$. It can be shown that $\boldsymbol{B}$ is identical in structure to the field of an magnetic dipole.
(d) A circular loop of wire, with radius R, lies in the xy plane (centered at the origin) and carries a current I running counterclockwise as viewed from the positive z axis. Using the result in (c), what is the (approximate) magnetic field at points far from the origin?
(e) Using the result in (c), find the the field for far off points on the z axis.

Problem 8



Find the magnetic field at point P on the axis of a tightly wound solenoid (helical coil) consisting of n turns per unit length wrapped around a cylindrical tube of radius a and carrying current I (Figure). Express your answer in terms of θ_1 and θ_2. Consider the turns to be essentially circular. What is the field on the axis of an infinite solenoid (infinite in both directions)?